For Test 1:

1. (10 pts)
   Consider the sphere $S$ in space with an equation:
   \[ x^2 + y^2 + z^2 + 2x + 4y - 6z + 10 = 0. \]
   (a) Find the center and radius of the sphere $S$.
   (b) Compute the distance from the point $P(0, 0, 6)$ to the center of $S$.
   (c) Find the distance of the point on the sphere $S$ closest to $P$.

2. (10 pts)
   Given three points $A(2, 0, 0), B(0, 1, 0)$, and $C(0, 0, 3)$.
   (a) Find a normal direction of the plane passing thro’ $A, B$ and $C$.
   (b) Find an equation of the plane passing thro’ $A, B$ and $C$.
   (c) Find the distance from the origin $(0, 0, 0)$ to the plane passing thro’ $A, B$ and $C$.

3. (10 pts)
   (a) Find the parametric equations of the straight line $L_1$ passing thro’ the origin $(0, 0, 0)$ and the point $(3, 2, 1)$.
   (b) Find the direction of the line $L_2$ with the parametric equations: $x = 10 - t, y = 8 - 2t, z = 2 + t$.
   (c) Find the intersection point of the straight lines $L_1$ and $L_2$.
   (d) Find the cosine of the angle between the straight lines $L_1$ and $L_2$.

4. (10 pts)
   A particle begins its motion at the location $(8, 1, -2)$ with the velocity vector function
   \[ \mathbf{v}(t) = (e^t, \frac{1}{1+t}, t^2). \]
   (a) Find its acceleration vector at the beginning.
   (b) Find its speed at $t = 0$.
   (c) Find its position vector function $\mathbf{r}(t)$.
For Test 2:

5. (10 pts)
Consider the function \( f(x, y) = x^2 \ln(x^2 + y^2) \).

(a) Compute all first partial derivatives of \( f(x, y) \).
(b) Find the linear approximation \( L(x, y) \) of \( f(x, y) \) at \((1, 0)\).
(c) Use \( L(x, y) \) to approximate the numerical value of \( f(0.99, 0.01) \).

6. (10 pts)
Let \( f(x, y) = x^2 y + x + y^2 \).

(a) Find the gradient vector \( \nabla f(1, 3) \).
(b) Find the directional derivative \( D_u f(1, 3) \) where \( u = \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle \).
(c) Explain why there is NO unit vector \( v \) such that \( D_v f(1, 3) = 10 \).

7. (10 pts)
Consider the function \( f(x, y) = (x^2 + y)e^y \).

(a) There is only one critical point for \( f(x, y) \). Find the critical point.
(b) Use the Second Derivative Test to classify the critical point.
For Test 3:

8. (10 pts)
Consider the double integral \( I = \int \int_R x^2 - y \, dA \) where \( R = [2, 6] \times [1, 5] \).

(a) Use a Riemann sum to estimate the actual value of \( I \) with \( m = 2 \), \( n = 2 \), and take the sample points to be the mid-point of the subrectangles.

(b) Use Fubini’s Theorem to compute the actual value of \( I \).

(c) Compare the answers of (a) and (b), and conclude whether the Riemann sum in (a) is an over-estimate or under-estimate of \( I \).

Answer: ______________________

9. (10 pts)

(a) Fill in the blank.
Let \( R \) be a region on the plane and \( f(x, y) \geq 0 \) on \( R \). Then the double integral \( \int \int_R f(x, y) \, dA \) can be interpreted as the ________________ under the surface \( z = f(x, y) \) over \( R \).

(b) Let \( D \) be the triangle with vertices \((0, 1), (-1, 0), \) and \((1, 0)\). Give an algebraic description of \( D \) as type II.

(c) Compute the double integral \( \int \int_D x^2 y \, dA \) where \( D \) is the region described in (b).

10. (10 pts)

(a) Compute the iterated integral \( \int_{-1}^2 \int_2^3 \int_0^1 xy^2 z^3 \, dz \, dy \, dx \).
[ Show all steps for full credit. ]

(b) Let \( E \) be the solid under the surface \( z = \sqrt{4 - x^2 - y^2} \) and over the square \([0, 1] \times [0, 1]\) on the \( xy \)-plane. Use Fubini’s Theorem to convert the triple integral \( \int \int \int_E \sin(xyz) \, dV \) into an iterated single integral.
[ Do not evaluate the iterated integral. ]