The 1988 Department of Mathematical Sciences Lecture Series
The Johns Hopkins University

PRELIMINARY PROGRAM FOR THOMPSON LECTURES

The lectures will be in Shaffer Hall, Room 3. Registration will be in Maryland Hall, Room 226, which will also be the place where mid-morning and mid-afternoon refreshment breaks will be held; participants may use this room at other times for discussions.

Sunday, June 19	
12-9 PM	Registration, Maryland Hall Room 226
Monday, June 20	
9:00	Opening remarks
9:15	Lecture I: The Exponential Function
10:15	Mid-morning break, Refreshments in Maryland 226
11:00	Contributed talks/Problems
2:00	Lecture II: The Triangle Inequality
3:00	Mid-afternoon break
3:45	Contributed talks/Problems
6:00	Swim party (bring your swimsuit!!) & cookout at Horn home
Tuesday, June 21	
9:15	Lecture III: Invariant Factors
10:15	Mid-morning break
11:00	Contributed talks/Problems
2:00	Lecture IV: Diagonal Elements, Singular Values, Majorization, Reflection Groups, and Elementary Divisors
3:00	Mid-afternoon break
3:45	Contributed talks/Problems
Wednesday, June 22	
9:00	Lecture V: The Shubert Calculus and Spectral Inequalities
10:00	Mid-morning break
10:45	Lecture VI: Spectrum of a Sum of Hermitian Matrices and Singular Values of a Product of General Matrices
afternoon free!	
Thursday, June 23	
9:15	Lecture VII: Submatrices
10:15	Mid-morning break
11:00	Contributed talks/Problems
2:00	Lecture VIII: Miscellaneous Inequalities
3:00	Mid-afternoon break
3:45	Contributed talks/Problems
Friday, June 24	
9:00	Lecture IX: Integral Quaternion Matrices
10:00	Mid-morning break
10:45	Lecture X: The Exponential Function, the Matrix-Valued Norm, the Matrix-Valued Numerical Range
12:15	Farewell luncheon at Johns Hopkins Club

Abstracts

Robert C Thompson

The lectures will focus principally on questions on which I have worked, with an attempt to bring the results into a broader perspective. I will generally take a low approach, and only hint at the high levels in which the same results can be viewed, and probably better understood. Open questions will be mentioned.

I have prepared lecture notes, generally 12-16 pages, which will be distributed at each lecture.

Lecture 1. The exponential function

Let x and y be noncommutative. Then there exists an element z such that $e^z = e^x e^y$. Several presentations of z will be discussed, including a classical one due to Dynkin, and one due to myself and F. Rouviere. The first terms with explicit numerical coefficients for the infinite series presentations of z will be shown. A conjecture based on computer evidence will be described. An approach through spectral estimation will be mentioned.

Lecture 2. The triangle inequality.

The matrix valued triangle inequality was discovered by me some years ago. It will be discussed, as well as its p-adic analogue. The matrix valued norm inequality will also be examined, including strategies for its proof, both real and p-adic versions.

Lecture 3. Invariant factors.

The invariant factors of algebraic combinations of matrices will be examined, particularly for integral matrices, although there will be brief mention of similarity invariant factors for matrices over a field. Over a principal ideal domain, the connection with Young tableaux will be outlined. It will appear, in this and some other lectures, that the deepest understanding of matrix spectral theory often becomes combinatorial.

Lecture 4. Diagonal elements, singular values, majorization, reflection groups, elementary divisors.

A major theme of this lecture will be to point out the role of finite reflection groups in the study of some classical topics in which majorization appears. Various "quasi-doubly-stochastic" transforms of a vector will be mentioned.

Lecture 5. The Schubert calculus and spectral inequalities.

The theme of this lecture is: Can spectral inequalities be understood in terms of geometry? This is in spite of the combinatorial basis that so often occurs. Specific attention will be paid to the spectral inequalities for a sum of Hermitian matrices.

Lecture 6. Spectrum of a sum of Hermitian matrices and the singular values of a product of general matrices.

The history of the problem of describing the allowable inequalities for the spectrum of a sum of Hermitian matrices will be described. Current knowledge will be summarized, to the extent that I know it. A discussion of the announced solution by B. V. Lidksii will also be given. Connections with tableaux will also be mentioned. Lie theoretic extensions will be pointed out.

Lecture 7. Submatrices.

The first, second, and third interlacing principles will be described. The Aronszajn inequality for the spectrum of complementary submatrices in a Hermitian matrix will be discussed, including its relation to the problem fo the spectrum of a sum of Hermitian matrices. The analogue for positive definite matrices extending the Fischer inequality will be mentioned. Some other inequalities involving spectra of submatrices will be mentioned.

Lecture 8. Miscellanious inequalities.

There will be several types of inequalities, apparently quite distinct, but exhibiting a somewhat common theme. Two spectral inequalities which are theorems only when the matrix dimensions are odd will be described. The connection between the 2×2 inversion rule and invariant factors will be mentioned briefly. The Ostrowski-Taussky determinantal inequality will appear. Least common left divisors and greatest common right multiples will present an appearance. So will the law of inertia, in real and p-adic forms. Interlacing and submatrices of complex symmetric or skew matrices also will occur. Some other types may appear.

Lecture 9. Integral matrices over more general domains.

Most of the lecture will be concerned with the Smith invariants of matrices with integral quaternon elements. However, there will also be a brief discussion of Smith invariants for matrices of algebraic integers, and of the first, second, and third interlacing principles for matrices with real (rather than integral) quaternion elements.

Lecture 10. Convergence domains for the Campbell-Baker-Hausdorff formula.

For the exponent $z = \log(e^x e^y)$ in Lecture 1, given in one of its several possible infinite series forms, when does the series converge if matrices are substituted for x and y? Some recent results will be described, including one based on a computer calculation.

I hope you will find something of value in the lectures.