1. Let \( \{X_t, t = 0, 1, 2, \ldots \} \) be a Markov chain with three states \( S = \{1, 2, 3\} \), initial distribution \( \pi = (0.2, 0.3, 0.5) \) and transition probability matrix

\[
P = \begin{pmatrix}
0 & 0 & 1 \\
0.5 & 0.3 & 0.2 \\
0 & 0.8 & 0.2
\end{pmatrix}
\]

(a) Find \( P(X_{t+2} = 1, X_{t+1} = 2|X_t = 3) \).
(b) Find the two step transition probability matrix \( P^{(2)} \) and specifically

\[
P(X_{t+2} = 2|X_t = 2)
\]

(c) Find \( P(X_2 = 1) \).
(d) Find \( E[X_1] \).

2. Consider the following (simple) epidemic model: A population of size \( N \) consists of infected and susceptible individuals. During each time period, each of the \( \binom{N}{2} \) possible pairs in the population will come in contact with probability \( p \). If a pair is in contact and one person in the pair is infected and the other susceptible, then the disease will be transmitted to the infected person. Nobody is ever cured of the disease.

(a) If there are \( k \) \( (k < N) \) infected individuals at time \( t \) in the population, what is the probability that a specified susceptible person will become infected in the period \( t \to t+1 \)?
(b) Define \( \{X(t) = \text{number of infected individuals at time step } t, t = 0, 1, 2, \ldots\} \). Is \( \{X(t), t = 0, 1, 2, \ldots\} \) a Markov chain?
(c) Write down the state space \( S \) for a small population \( (N = 5) \) with one infected individual at time \( t = 0 \) and specify the transition probability matrix for this case.

3. Ants march single file in a row. The ants either carry leaves (for building nests) or food. Three out of four ants carrying leaves are followed by an ant carrying food, while only one out of five ants carrying food are followed by an ant carrying leaves. What proportion of ants carry leaves?