Purpose: Solve a homogeneous system to find equilibrium prices for an exchange model economy.

Prerequisite: Section 1.5

MATLAB functions used: 
- /, eye, sum; and econdat and rref from Lay's Toolbox

1. (hand) Let 
\[ T = \begin{bmatrix} .20 & .17 & .25 & .20 & .10 \\ .25 & .20 & .10 & .30 & 0 \\ .05 & .20 & .10 & .15 & .10 \\ .10 & .28 & .40 & .20 & 0 \\ .40 & .15 & .15 & .15 & .80 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \]

and consider the system of linear equations \( Tx = x \).

(a) Write out the five equations in this system:

(b) Collect terms in your equations to get a homogenous linear system, and write out the five new equations:

2. Let \( Bx = 0 \) denote the homogenous system you obtained in 1(b), and calculate the reduced echelon form of \([B \ 0]\). Record the reduced form below. These lines will get the matrix and do the calculation:

\[ \text{econdat} \]
\[ \text{rref([B zeros(5,1)])} \]
3. (hand) First read about Leontief Economic Models in Section 1.5 of the text. Now consider an exchange model economy which has five sectors, Chemicals, Metals, Fuels, Power, and Agriculture; and assume the matrix \( T \) in question 1 above gives an exchange table for this economy as follows:

\[
T = \begin{bmatrix}
0.20 & 0.17 & 0.25 & 0.20 & 0.10 \\
0.25 & 0.20 & 0.10 & 0.30 & 0 \\
0.05 & 0.20 & 0.15 & 0.10 & 0 \\
0.10 & 0.28 & 0.40 & 0.20 & 0 \\
0.40 & 0.15 & 0.15 & 0.15 & 0.80
\end{bmatrix}
\]

Notice that each column of \( T \) sums to one, indicating that all output of each sector is distributed among the five sectors, as should be the case in an exchange economy. The system of equations \( Tx = x \) must be satisfied for the economy to be in equilibrium. As you saw above, this is equivalent to the system \( Bx = 0 \).

(a) Let \( x_C \) represent the value of the output of Chemicals, \( x_M \) the value of the output of Metals, etc. Using the reduced echelon form of \( [B \ 0] \) from question 2, write the general solution for \( Tx = x \):

\[
\begin{bmatrix}
x_C \\
x_M \\
x_F \\
x_P \\
x_A
\end{bmatrix} = \begin{bmatrix}
\end{bmatrix}
\]

(b) Suppose that the economy described above is in equilibrium and \( x_A = 100 \) million dollars. Calculate the values of the outputs of the other sectors and record this particular solution for the system \( Tx = x \):

\[
\begin{bmatrix}
x_C \\
x_M \\
x_F \\
x_P \\
x_A
\end{bmatrix} = \begin{bmatrix}
\end{bmatrix}
\]

4. (hand) Consider the matrices \( T \) and \( B \) created above. As already observed, each column of \( T \) sums to one. Consider how you obtained \( B \) from \( T \) and explain why each column of \( B \) must sum to zero.

5. (Extra credit; attach paper) Let \( B \) be any matrix of any shape, with the property that each column of \( B \) sums to zero. Explain why the reduced echelon form of \( B \) must have a row of zeros.