Purpose: To count the number of arithmetic operations done in certain matrix products and see that the placing of parentheses can make a difference in some products.

Prerequisite: Section 2.1

MATLAB functions used: * , rand, flops

NOTE: A version of MATLAB before Version 6 is needed for this project. Version 6 does not have flops.

Definition. A floating point operation, or "flop" for short, is one addition, subtraction, multiplication or division of two real numbers, done with floating point arithmetic. It is customary to say "addition" to mean addition or subtraction and "multiplication" to mean multiplication or division.

In all versions of MATLAB before Version 6, there is a function called flops which counts how many flops are performed in a session. You can see its value at any time by typing flops. It can be used another way: you can set its value to zero by typing flops(0), and then it counts the number of floating point operations done after that.

1. (hand) Let \( x \) and \( y \) denote \( nx1 \) matrices. Explain why, in the (row)(column) product \( x^T y \), the number of additions is about the same as the number of multiplications. Use this to explain why the number of additions is about the same as the number of multiplications in the product \( XY \) of any two matrices (where \( X \) and \( Y \) denote matrices of any sizes that can be multiplied).

2. This is similar to Exercise 9, Section 2.8. You will see that the number of flops done to calculate \( ABD \) can be greatly different, depending on whether you calculate \( AB \) first, or \( BD \). Record the number of flops for each choice of \( D \) in the table below.

   The following lines will get you started. We use semicolons after the matrices involving \( D \) because they are quite large. If you want to see those matrices, type commas instead of semicolons:

   \[
   A = \text{rand}(2, 2), \quad B = \text{rand}(2, 2), \quad D = \text{rand}(2, 200);
   \]

   \[
   \text{flops}(0), \quad (A*B)*D; \quad \text{flops(0), A*(B*D); flops}
   \]

   Repeat the commands using \( D = \text{rand}(2, 300) \) and then \( D = \text{rand}(2, 400) \). You can avoid some retyping by using the up arrow key.

   Count of Flops when \( D \) is \( 2xm \)

   \[
   \begin{array}{cccc}
   \text{Size of } D & 2x200 & 2x300 & 2x400 \\
   \hline
   A*(B*D) & \text{-----------------------------} & \text{-----------------------------} & \text{-----------------------------} \\
   (A*B)*D & \text{-----------------------------} & \text{-----------------------------} & \text{-----------------------------} \\
   \end{array}
   \]

3. (hand) Compare your counts above, when \( m = 200 \), with the answer to Exercise 9, Section 2.8. Explain why your counts here are twice as big as the ones given there.
4. Modify the lines typed in question 2 to do the same calculations for square matrices, of sizes 2x2, 5x5 and 10x10. Record your results:

<table>
<thead>
<tr>
<th>Size of matrices</th>
<th>2x2</th>
<th>5x5</th>
<th>10x10</th>
</tr>
</thead>
<tbody>
<tr>
<td>A*(B*D)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A*B)*D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. (hand) Explain why rearranging parentheses made a difference in question 2 but no difference in question 4.