Purpose: To study a linear system model of an open sector economy.

Prerequisites: Sections 2.1, 2.7

MATLAB functions used: \texttt{rref}, \texttt{sum}, \texttt{eye}; and Lay's Toolbox

Background. This is based on Exercise 13 in Section 2.7, where an economy with 7 production sectors is described. Each sector produces goods and each uses some of the output of the other sectors. There is also an open sector, i.e., a sector which only consumes. The consumption matrix will be called $C$, and $d$ will denote the demand vector for the open sector. When $C$ and $d$ are given, a solution $x$ to the equation $x = Cx + d$ is called a production vector. If there is a solution $x$ with all entries nonnegative, that means this economy is possible, i.e., it could exist. Notice the equation $x = Cx + d$ can be rewritten as $(I - C)x = d$.

When the matrix $C$ has each entry nonnegative and each column sum less than one, Theorem 11 guarantees that $I - C$ will be invertible and that the economy is possible for any nonnegative demand vector $d$ -- that is, the unique vector $x$ which satisfies $(I - C)x = d$ will also be nonnegative. (The proof of Theorem 11 shows why: for such $C$, all entries of the matrix $(I - C)^{-1}$ are nonnegative, so when $d$ has nonnegative entries, $x = (I - C)^{-1}d$ must also have nonnegative entries.)

1. (a) Type the following lines to get the data for $C$ and $d$ and to calculate the column sums of $C$. Inspect to be sure each entry of $C$ and $d$ is nonnegative and that each column sum of $C$ is less than one:

\begin{verbatim}
c2s7
13
sum(C)  \quad (\text{sum(C)} \text{ yields a row vector containing the sum of each column})
\end{verbatim}

Type the following lines to create $I - C$ and to solve the equation $(I - C)x = d$.

\begin{verbatim}
M = eye(7) - C  \quad (\text{eye(7)} \text{ creates a 7x7 identity matrix})
R = rref( [M d] ), x = R(:, 8)  \quad (\text{R(:, 8)} \text{ is column 8 of the matrix R})
\end{verbatim}

Record $d$ and $x$ below. Notice the display of $x$ has the expression $1.0e+005 *$ above a column of numbers. Don't ignore that! It means each number in the column is multiplied by $10^5$.

Choose two more nonnegative demand vectors $d_1$ and $d_2$ and solve for the production vector for each. Record all these vectors.

\begin{verbatim}
d = \quad x = \quad d_1 = \quad x_1 = \quad d_2 = \quad x_2 =
\end{verbatim}

(b) Discuss: why does it matter that the entries of the solution $x$ should be nonnegative, in an economic model like the above?

2. (a) Experiment to increase the (1,1) entry of $C$, until you find a new consumption matrix that gives a solution with some negative entries -- that is, until solving $(I - C)x = d$ yields some negative entries in $x$. Record your results:

\begin{verbatim}
New value of the (1,1) entry of $C$: $x =
\end{verbatim}

(b) Discuss (use back of paper): What do the numbers in your new matrix $C$ say about the economy? That is, why does it seem reasonable that your new matrix is not a valid consumption matrix?