SIF Computer Project: Traffic Flow

Name(s) _______________________________

to accompany Spence, Insel & Friedberg’s *Elementary Linear Algebra*.

Instructions: It will be easiest to use MATLAB or a calculator with matrix functions, to do the calculations. If you work with a partner, put both names above and turn in one solution.

Description: This project is based on the example “Analysis of Traffic Flow” in Section 2.2 of our text, *Elementary Linear Algebra*, by Spence, Insel and Friedberg. To begin, carefully study Figure 2.3 and the discussion of this example in the text. Notice that each $y_i$, $z_i$ and $w_i$ represents the number of vehicles on a certain section of street between two particular intersections. Be sure you understand how the equation for each $y_i$, $z_i$ and $w_i$ was obtained.

The city engineer knows that traffic moves more slowly through $P_9$ than other intersections, so she is not surprised to see that the solution shows $w_3 = 1356$, which is very large compared to $w_1 = 698$, $w_2 = 365$ and $w_4 = 581$.

Here you will analyze three possible ways to reduce the value of $w_3$. Assume the initial flow values into the city are as in the text, that is, let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \end{bmatrix}$.

1. One possibility is to widen the street $P_2 \rightarrow P_5$; she thinks that the new flow rates leaving $P_2$ would be as shown on the right:

(i) Write equations for the new $y_i$, $z_i$ and $w_i$ values. (Only some will really be new.)

(ii) Using your equations, write matrices $A_1$, $B_1$ and $C_1$ so that $y = A_1x$, $z = B_1y$, and $w = C_1z$ are true:

(iii) Calculate the matrix $C_1B_1A_1$ and the vector $w = C_1B_1A_1x$. Record these:

2. A different approach would be for the city to widen the street
\( P_6 \rightarrow P_5 \), and she thinks the new flow rates out of \( P_6 \) would be:

(i) Write equations for the new \( y_i \), \( z_i \) and \( w_i \) values. (Only some will really be new.)

(ii) As in question 1(ii), write matrices \( A_2 \), \( B_2 \) and \( C_2 \) associated with your equations:

(iii) As in 1(iii), calculate the matrix \( C_2B_2A_2 \), new vector \( \mathbf{w} = C_2B_2A_2\mathbf{x} \), and record these:
3. A third alternative is for the city to try to attract Ztron Electronics to open a plant at \( P_6 \), by offering low property taxes for five years. This would cost the city about the same amount as one of the road widening jobs described above. If Ztron does open a plant at \( P_6 \), they estimate that 500 of the cars arriving at \( P_6 \) would stay there. So the new number of cars on \( P_6 \rightarrow P_5 \) would be \( 0.6(y_3 - 500) \) and the new number on \( P_6 \rightarrow P_9 \) would be \( 0.4(y_3 - 500) \).

(i) Write an equation for each \( z_i \):

(ii) Calculate the new \( w \) vector, using matrices and vectors. Show enough detail that it is clear how you calculated it, and record its value:

4. If the city can afford only one of the actions described above, what do you recommend and why?