Math 211A Second Homework

1. Let $C$ be the curve $x^4 + y^3 = xy$ in a complex affine plane $E$.
   (i) Find the multiple points of $C$.
   (ii) Find the asymptotic directions (lines) of $C$.
   (iii) Check if the intersection points of the asymptotic directions of $C$ and the projective closure $\hat{C}$ of $C$ are single (the intersection is taken in $\hat{E}$), i.e. if the intersection multiplicities are one.

2. Solve problem 1 for the curve $x^2 + y^3 = z^4$ in a complex three dimensional affine space $E$.

3. (The last case of Desargues’ Theorem) Let $P$ be a projective space, and $D$, $D'$, and $D''$ be three coplanar lines in $P$ (i.e. $\nu(D \cup D' \cup D'')$ is a two dimensional projective plane) that have a common point $O$. Let $A, B \in D$, $A', B' \in D'$, and $A'', B'' \in D''$, be points distinct from one another and from $O$, and let $I = D_{AA'} \cap D_{BB'}$, $J = D_{AA''} \cap D_{BB''}$, and $K = D_{A'A''} \cap D_{B'B''}$. If $O$ lies on the line through $I$ and $J$ prove that $I$, $J$, and $K$ are collinear.