Exercises (to be done but not turned in): 6.2-6.5, 6.7-6.8, 6.10, 6.12-6.14; 7.3, 7.5. Problems to be turned in: All numbers refer to problems in the Yellow and Blue Book.

1. Let

\[ A = \{(v, w, x, y, z) \in \mathbb{R}^5 \mid x = 0\}, \]
\[ B = \{(v, w, x, y, z) \in \mathbb{R}^5 \mid y = 0\}, \]
\[ C = \{(v, w, 0, 0, z) \mid v, w, z \in \mathbb{R}\}. \]

Prove that \( A \cap B = C \).

2. Consider the sets

\[ A = \{(x, y) \in \mathbb{R}^2 \mid y \geq x^2 + 1\}, \]
\[ B = \{(x, y) \in \mathbb{R}^2 \mid y \leq -x^2 - 1\}, \]
\[ C = \{(x, y) \in \mathbb{R}^2 \mid |y| \geq 2|x|\}, \]
\[ D = A \cup B. \]

Does \( C = D \)? Is one of \( C \) and \( D \) a subset of the other, or is neither a subset of the other? Carefully state and prove your answer. (Suggestion: \( x^2 = |x|^2 \).)

3. For any integer \( n \), define

\[ n\mathbb{Z} = \{a \in \mathbb{Z} \mid a = nk \text{ for some } k \in \mathbb{Z}\}. \]  \hspace{1cm} (1)

Recall that if \( a, b \in \mathbb{Z} \), to say that \( a \) divides \( b \) means that \( b = ac \) for some \( c \in \mathbb{Z} \). Exactly one of the following two statements is true for all \( a, b \in \mathbb{Z} \):

- If \( a \) divides \( b \), then \( a\mathbb{Z} \subseteq b\mathbb{Z} \).
- If \( a \) divides \( b \), then \( b\mathbb{Z} \subseteq a\mathbb{Z} \).

Determine which statement is always true, and prove it.

4. 7.1(b).

5. 7.1(e).

6. 7.5(a).

7. 7.14. (For (a), suppose \( (A \setminus B) \cap B \neq \emptyset \), and proceed by contradiction.)