Exercises (to be done but not turned in): 12.1, 12.4, 12.5, 12.6, 12.8, 12.9.
Problems to be turned in: All numbers refer to problems in the Yellow and Blue Book.

1. Let $S$ be a nonempty subset of $\mathbb{R}$, and let $U$ be an upper bound for $S$. Prove that the following are equivalent:
   - (Sup) $U = \sup S$.
   - (Arbitrarily close) For any $\epsilon > 0$, there exists some $x \in S$ such that $U - \epsilon < x$.

   Suggestion: Instead of proving that (Arbitrarily close) implies (Sup), you may find it easier to prove the contrapositive.

2. Let $S$ be the open interval $(-7, -2)$. Determine (guess) the value of $\sup S$, and prove that your guess is correct.

3. Let $S = \left\{ \frac{7n}{5n - 2} \mid n \in \mathbb{Z}, n > 0 \right\}$. Determine (guess) the value of $\inf S$, and prove that your guess is correct.

4. Let $S = (2, \pi^2) \cap \mathbb{Q}$. Determine (guess) the value of $\sup S$, and prove that your guess is correct.

5. 12.8.

6. 12.13(a,b,c). (I.e., skip part (d).)

7. (a) Is it possible that there exists a real number $M$ such that $r \leq M$ for all rational numbers $r$? Prove or disprove.

   (b) Is it possible that there exists a real number $m > 0$ such that $r \geq m$ for all positive rational numbers $r$? Prove or disprove.