Definitions: Let \( f : X \to Y \) be a function, let \( A \) be a subset of \( X \), and let \( B \) be a subset of \( Y \).

restriction We define the restriction of \( f \) to \( A \) to be the function \( f|_A : A \to Y \) defined by \( f|_A(x) = f(x) \) for all \( x \in A \).

co-restriction If it happens to be the case that for all \( x \in X \), we have \( f(x) \in B \), we define the co-restriction of \( f \) to \( B \) to be the function \( f|_B : X \to B \) given by \( f|_B(x) = f(x) \) for all \( x \in X \).

bi-restriction If it happens to be the case that for all \( x \in A \), we have \( f(x) \in B \), we define the bi-restriction of \( f \) to \( A, B \) to be the function \( f|_{A,B} : A \to B \) given by \( f|_{A,B}(x) = f(x) \) for all \( x \in A \).

Exercises (to be done but not turned in): 14.1, 14.2, 14.3, 14.4, 14.6, 14.9, 15.4, 15.5, 15.8, 15.9.

Problems to be turned in: All numbers refer to problems in the Yellow and Blue Book. You will also need to use the definition of composite function from chapter 16 (p. 167).

1. Define a function \( g : Z \to Z \times Z \) by the formula \( g(x + y) = (x, y) \) for all \( x, y \in Z \). Is \( g \) well-defined? Prove your answer.

2. Complete (in as interesting a manner as possible) and prove the following theorem: Let \( A, B, \) and \( Y \) be sets, and let \( f : A \to Y \) and \( g : B \to Y \) be well-defined functions. Then the formula
   \[
   h(x) = \begin{cases} f(x) & \text{if } x \in A, \\ g(x) & \text{if } x \in B, \end{cases}
   \]
gives a well-defined function \( h : (A \cup B) \to Y \) if and only if (condition on \( f \) and \( g \)).

3. Let \( f \) and \( g \) be functions from \( R \) to \( R \).
   (a) If \( f(x) = g(x) \) for infinitely many \( x \in R \), is it necessarily the case that \( f = g \)? Prove or give a counterexample.
   (b) If \( f(x) = g(x) \) for all but finitely many \( x \in R \), is it necessarily the case that \( f = g \)? Prove or give a counterexample.

4. 14.16.

(Cont. on next page.)
5. (a) 15.19(a).
(b) For any function $f : X \to Y$ and any $A \subseteq X$, define

$$f(A) = \{ y \in Y \mid y = f(a) \text{ for some } a \in A \}.$$  

Prove the **Embedding Lemma**: If $f : X \to Y$ is one-to-one and $A \subseteq X$, then the function $g : A \to f(A)$ given by $g(a) = f(a)$ for all $a \in A$ is a bijection.

6. 15.23.

7. (a) 15.26(a).
(b) 15.26(c). (You may assume the results of 15.26(b) here.)