Problems to be done but not turned in: 5.1, 5.3, 5.5, 5.7, 7.1, 7.3, 7.5.
Problems to be turned in: All numbers refer to exercises in Ross.

1. Suppose $S$ and $T$ are bounded subsets of the real numbers and $U = S \cap T$ is nonempty.
   (a) Prove that both $\sup S$ and $\sup T$ are upper bounds for $U$.
   (b) Prove that $\sup U \leq \min\{\sup S, \sup T\}$.

2. We define an interval in $\mathbb{R}$ to be a nonempty subset $S \subseteq \mathbb{R}$ such that if $x, y \in S$ and $x < z < y$, then $z \in S$.
   (a) Give an example of a nonempty $T \subseteq \mathbb{R}$ that is not an interval, and prove that $T$ is not an interval.
   (b) Now suppose $S$ is a bounded interval, let $a = \inf S$, and let $b = \sup S$. (Note that $a$ and $b$ need not be elements of $S$.) Prove that if $a < x < b$, then $x \in S$.

3. Let $S$ be a nonempty set of positive real numbers such that $\sup S = +\infty$, and let $T = \left\{ \frac{1}{x} \mid x \in S \right\}$. Prove that $\inf T = 0$.

4. Ex. 7.2.

5. Ex. 7.4.

6. Guess the value of $\lim \frac{2n + 1}{n^2 - 5}$, and prove your answer, using the definition of limit.

7. Guess the value of $\lim \frac{5n + 8}{2n - 7}$, and prove your answer, using the definition of limit.