1. (10 points) Problem omitted, as it requires you to recite a definition from Sect. 10. Your exam will still begin by requiring you to recite a definition or theorem, just not one from Sect. 10.

2. (10 points) Suppose \(a_n, b_n,\) and \(c_n\) are sequences such that \(\lim a_n = 5,\) \(\lim b_n = 7,\) and \(\lim c_n = -11.\) Determine the value of

\[
\lim(a_n(b_n + c_n))
\]

and carefully use the limit laws to justify (prove) your answer. (In particular, each time you use a limit law, state which limit law you are using.)

In questions 3–5, you are given a statement. If the statement is true, you need only write “True”, though a justification may earn you partial credit if the correct answer is “False”. If the statement is false, write “False”, and justify your answer as specifically as possible. (Do not just write “T” or “F”, as you may not receive any credit; write out the entire word “True” or “False”.)

3. (12 points) TRUE/FALSE: If \(S\) is a nonempty bounded subset of \(\mathbb{R},\) and \(\inf S = 13,\) then it must be the case that \(13 \in S.\)

4. (12 points) TRUE/FALSE: It is possible that there exists a convergent sequence \(s_n\) such that \(s_n\) is irrational for all \(n \in \mathbb{N}\) and \(\lim s_n\) is rational.

5. (12 points) TRUE/FALSE: If \(S\) is a nonempty bounded subset of \(\mathbb{R},\) and \(x \leq 10\) for all \(x \in S,\) then it must be the case that \(\sup S = 10.\)

6. (14 points) PROOF QUESTION. Let \(S = \{x \in \mathbb{Q} | x > 2\}.\) (Note that \(S\) contains only rational numbers.) Prove that \(\inf S = 2.\)

7. (14 points) PROOF QUESTION. Use the definition of limit to prove that

\[
\lim \frac{n}{n^2 + 1} = 0.
\]

8. (16 points) PROOF QUESTION. Suppose \(s_n\) is a sequence such that \(\lim s_n = 5.\)

(a) State the definition of what it means to say that \(\lim s_n = 5.\)

(b) Prove that there exists a real number \(N\) such that if \(n \in \mathbb{Z}\) and \(n > N,\) then \(s_n < 7.\)