Name

1. a. Solve
\[ \frac{dy}{dx} = \frac{1 - y^2}{y} \]

\[ \frac{y}{1 - y^2} dy = dx. \] Integrate to get \(-1/2 \ln |1 - y^2| = x + C\) Impossible to solve for \(y\).

b. Give the solution that satisfies the initial condition \(y(0) = 2\).

Plug in \(-1/2 \ln 3 = C\)

2. For which, if any, of the following two IVP’s does the theorem we studied imply a unique solution?
   a. \(\frac{dx}{dt} + \cos x = \sin t, x(\pi) = 0\)
   b. \(\frac{dy}{dx} = 3x - \sqrt[3]{y - 1}, y(2) = 1\)

To apply theorem 1 to (a), let \(f(x, t) = \sin t - \cos x\). Then both \(f\) and \(\frac{\partial f}{\partial x}\) are continuous everywhere, so (a) has a unique solution.

For (b), \(f(x, y) = 3x - \sqrt[3]{y - 1}\), for this \(f\), \(\frac{\partial f}{\partial y}\) is discontinuous at \((2, 1)\), so Theorem 1 does not apply.

3. Given the IVP
\[ \frac{dy}{dx} = y - x, y(0) = 2 \]

use Euler’s method to estimate \(y(2)\) using the stepsize \(\Delta x = 1\).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>dy/dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

\(y(2) \approx 7\)
4. A brine solution of salt flows at a constant rate of 8 L/sec into a tank which initially held 50L of a brine solution in which was dissolved 2.0 kg of salt. The well-stirred mixture flows out of the tank at the same rate. The concentration of the brine entering the tank is 0.5 kg/L.

Set up, but do not solve, a differential equation for the amount of salt in the tank at time \( t \). Let \( A = A(t) \) be the amount of salt in kg in the tank at time \( t \).

\[
\frac{dA}{dt} = \text{rate in} \ - \text{rate out}.
\]

\[
\frac{dA}{dt} = 8 \times 0.5 \ - \ 8 \times \frac{A}{50}
\]

5. Solve \( \frac{dy}{dx} + 4y - e^{-x} = 0 \), \( y(0) = 2 \) by finding an integrating factor.

Integrating factor is \( e^{4x} \) Multiplying both sides gives

\[
e^{4x} \frac{dy}{dx} + 4ye^{4x} = e^{3x}
\]

or

\[
\frac{d}{dx} ye^{4x} = e^{3x}
\]

Integrate to get

\[
ye^{4x} = \frac{1}{3}e^{3x} + C \text{ and then}
\]

\[
y = \frac{1}{3}e^{-x} + Ce^{-4x} \text{ plug in:}
\]

\[
2 = \frac{1}{3} + C
\]

6. Solve the IVP

\[
y'' - 4y' + 3y = 0, \ y(0) = 1, \ y'(0) = \frac{1}{3}
\]

Solve \( m^2 - 4m + 3 \), getting \( m = -3 \) and \( m = -1 \)

General solution is \( y = C_1 e^{-t} + C_2 e^{-3t} \) Then \( y' = -C_1 e^{-t} - 3C_2 e^{-3t} \)

Plug in to get \( C_1 + C_2 = 1 \) and \(-C_1 - 3C_2 = \frac{1}{3}\)
7. Find the general solution to
\[ y'' + 4y = 0 \]
\[ y = C_1 \sin 2t + C_2 \cos 2t \]
or
\[ y = C_1 e^{2it} + C_2 e^{-2it} \]

8. Solve the exact equation
\[ (e^t x + 1)dt + (e^t - 1)dx = 0, \quad x(1) = 1 \]
The solution will be \( F(x, t) = C \), once we find \( F \).
\[ \frac{\partial F}{\partial x} = e^t - 1 \]
so
\[ F = (e^t - 1)x + \phi(t) \]
Now, \( \frac{\partial F}{\partial t} = xe^t + \phi'(t) = e^t x + 1 \), so \( \phi'(t) = 1 \) and \( \phi(t) = t \).
(Dropping the constant.
So, \( F = (e^t - 1)x + t \) and the general solution is
\( (e^t - 1)x + t = C \)
Plug in to get \( e = C \)