Gaussian Elimination Algorithms

Input: an \( n \times n \) matrix \( A \)
Output: the upper triangular part of \( A \) contains \( U \) in a factorization of \( PAQ = LU \),
the strictly lower triangular part of \( A \) contains the strictly lower triangular part of \( L \),
vector \( \lambda \): permutation matrix \( P \) moves row \( \lambda_i, \ i = 1, \ldots, n, \) of \( A \) to row \( i \) of \( PA \),
vector \( \omega \): permutation \( Q \) moves column \( \omega_j, \ j = 1, \ldots, n, \) of \( A \) to column \( j \) of \( AQ \)

### no pivoting (GENP):

for \( k = 1, \ldots, n - 1 \)
  PIVOTING: skip  \% zero column \( k \) below diagonal
  ELIMINATION:
  if ( \( a_{kk} \neq 0 \) )
    for \( i = k + 1, \ldots, n \)
      \( a_{ik} \leftarrow \ell_{ik} = a_{ik}/a_{kk} \)  \% calculate and store multipliers
      \( ( \leftarrow \) means overwrite in memory, \( \ell_{ik} \) is used to make the pseudocode easier to follow, but it is not stored separately)
    end
    for \( i = k + 1, \ldots, n \)
      for \( j = k + 1, \ldots, n \)
        \( a_{ij} \leftarrow a_{ij} - \ell_{ik} a_{kj} \)
      end
    end
  else
    Stop, algorithm fails when \( a_{kk} = 0 \)
  end
end

### partial pivoting (GEPP):

for \( k = 1, \ldots, n \)
  \( \lambda_k = k \)  \% initialize to remember pivoting
end
for \( k = 1, \ldots, n - 1 \)
  PIVOTING:
    find \( \rho \geq k \) such that \( |a_{pk}| \geq |a_{ik}| \) for \( i \geq k \)
    \( \lambda_k \leftarrow \lambda_{\rho} \)  \( (\leftarrow \) means switch \)
    for \( j = 1, \ldots, n \)
      \( a_{kj} \leftarrow a_{kj} \)
    end
  ELIMINATION:
  if ( \( a_{kk} \neq 0 \) )
    for \( i = k + 1, \ldots, n \)
      \( a_{ik} \leftarrow \ell_{ik} = a_{ik}/a_{kk} \)  \% calculate and store multipliers
    end
    for \( i = k + 1, \ldots, n \)
      for \( j = k + 1, \ldots, n \)
        \( a_{ij} \leftarrow a_{ij} - \ell_{ik} a_{kj} \)
      end
    end
  else
    Continue algorithm, column \( k \) already zero below diagonal
  end
end
complete pivoting (GECP):

for $k = 1, \ldots, n$
  $\lambda_k = k$
  $\omega_k = k$
end

for $k = 1, \ldots, n - 1$
  PIVOTING:
    Find $\rho \geq k$ and $\gamma \geq k$ such that
    $|a_{\rho,\gamma}| \geq |a_{i,j}|$ for all $i, j \geq k$
    $\lambda_k \leftrightarrow \lambda_{\rho}$ (\(\leftrightarrow\) means switch)
    $\omega_k \leftrightarrow \lambda_{\gamma}$
    for $j = 1, \ldots, n$
      $a_{k,j} \leftrightarrow a_{\rho,j}$
  end
  for $i = 1, \ldots, n$
    $a_{i,k} \leftrightarrow a_{i,\gamma}$
  end
  ELIMINATION:
    Same code as for partial pivoting
end

rook pivoting (GERP):

for $k = 1, \ldots, n$
  $\lambda_k = k$
  $\omega_k = k$
end

for $k = 1, \ldots, n - 1$
  PIVOTING:
    Find $\rho \geq k$ such that $|a_{\rho,k}| \geq |a_{i,k}|$ for $i \geq k$
    Find $\gamma \geq k$ such that $|a_{\rho,\gamma}| \geq |a_{p,j}|$ for $j \geq k$
    Find $\rho \geq k$ such that $|a_{\rho,\gamma}| \geq |a_{i,\gamma}|$ for $i \geq k$
    Find $\gamma \geq k$ such that $|a_{\rho,\gamma}| \geq |a_{p,j}|$ for $j \geq k$
    repeat until $\rho$ and $\gamma$ do not change
    $\lambda_k \leftrightarrow \lambda_{\rho}$ (\(\leftrightarrow\) means switch)
    $\omega_k \leftrightarrow \lambda_{\gamma}$
    for $j = 1, \ldots, n$
      $a_{k,j} \leftrightarrow a_{\rho,j}$
  end
  for $i = 1, \ldots, n$
    $a_{i,k} \leftrightarrow a_{i,\gamma}$
  end
  ELIMINATION:
    Same code as for partial pivoting
end