## NAME

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>175</strong></td>
<td></td>
</tr>
</tbody>
</table>
Show all work.
1. Compute the following integrals. In this problem you must, if necessary, reduce to one or more of the basic 18 integrals. Calculator answers alone are worth only 1 point.
(48 points, a,b,c are 6 each, d,e,f are 10 each)

\[
a) \int \frac{x^2 - 1}{x^2 + 1} \, dx
\]

\[
b) \int_{-2}^{1} |x + 1| \, dx
\]

\[
c) \int \frac{x}{x^4 + 1} \, dx
\]
d) \[ \int x \ln x \, dx \]

e) \[ \int \sin^5 x \cos^2 x \, dx \]

f) \[ \int \frac{x + 1}{x^4 + 2x^3} \, dx \]
2. Compute the following improper integrals, showing steps.
(12 points, 6 each)

a) \( \int_{1}^{\infty} \frac{1}{x^4} \, dx \)

b) \( \int_{0}^{1} (1 - x)^{-1.01} \, dx \)
3. Compute and graph the region  
(16 points, a=6 points, b=10 points)  
   a) The area of the region bounded by the curves \( x = 1 - y^2 \) and \( x = y^2 \)  

   b) The volume generated by revolving the region bounded by \( y = 1 + x^2 \), \( x + y = 3 \), and \( x = 0 \) around the y axis. 

4. Graph and find the area inside \( r = 3 \sin 2\theta \). Include the formula you use for the area.  
(10 points)
5. Solve the differential equations
(14 points, 7 each)

a) \( y' = \frac{xy}{1 + x} \)

b) \( xy' - 2y = x^3 \)

6. Find the arc length of the curve \( y = x^2 - \frac{\ln x}{8} \) for \( 1 \leq x \leq 2 \).
(10 points)
7. Find the value the following sums converge to.
(12 points, 6 each)

a) \[ \sum_{n=0}^{\infty} \frac{1}{4^n} \]

b) \[ \sum_{n=1}^{\infty} \left( \frac{2}{n(n+2)} \right) \]

8. Determine whether the following series converge or diverge by using an appropriate test or argument. You MUST state the test you use and show all necessary work (except skip showing things decrease).
(25 points, 5 each)

a) \[ \sum_{n=1}^{\infty} \frac{n^3 + n}{n^4 + n^2} \]

b) \[ \sum_{n=1}^{\infty} \frac{e^n}{n} \]
c) \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} \)

d) \( \sum_{n=1}^{\infty} \sqrt{\frac{n + 1}{n^4 + 1}} \)

e) \( \sum_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)^n \)
9. Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Make sure to state the test or reason you use, and show that it applies (except skip showing things decrease). (8 points)

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{2n + 1} \]

10. Find the values of \( x \) where the following series converges. Be sure to check the endpoints. State the values of \( x \) where the series is conditionally convergent. (12 points)

\[ \sum_{n=0}^{\infty} \frac{(-1)^nx^n}{2^n n^2} \]
11. Compute the Maclaurin (Taylor) series around $a = 0$ up to the $x^3$ term for the following functions (12 points, 6 each)

a) $f(x) = x^2 e^x$

b) $f(x) = \frac{1}{(x + 1)^2}$