<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
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<td>1</td>
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<td>Total</td>
<td>86</td>
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Show all work.
1. Compute the following, showing all necessary steps. (15 points, 5 each)

   a) \[ \sum_{k=1}^{99} (k + 1)^2 \]

   b) \[ \sum_{k=1}^{10000} \left( 2^k - \frac{2^k}{2} \right) \]

   c) \[ \lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{3n + k}{n^2} \right) \]

2. Approximate \( \int_{0}^{2} \sin(\pi x) \, dx \) with 4 right rectangles. Draw a picture. Also compute the exact answer. (10 points)
Show all work. In doing integrals you must reduce each problem to one or more of the basic 18 integrals. Calculator answers alone are worth only 1 point.

3. Compute the following integrals.
(45 points, 5 each)

a) \[ \int \frac{x}{x + 1} \, dx \]

b) \[ \int x + \frac{1}{x} + x^2 \left( \frac{x}{2} + \frac{2}{x} \right) \, dx \]

c) \[ \int_1^2 \frac{x^2 + 1}{x^3} \, dx \]

d) \[ \int e^x \sec(e^x) \tan(e^x) \, dx \]
e) \( \int_{-1}^{1} |x^2 + 1| \, dx \)

f) \( \int_{-1}^{0} \sqrt{-x} \, dx \)

g) \( \int \sin^{100} x \cos x \, dx \)

h) \( \int \frac{1}{x(\ln x)^2} \, dx \)

i) \( \int \frac{x}{x^4 + 1} \, dx \)
4. Show (without using a calculator to numerically find the value of the integral) that
\[ \int_0^1 e^{x^2} \, dx \leq e. \]
(6 points)

5. Solve the following differential equations.
(10 points, 5 each)

a) \( y' = xy \)

b) \( y' = \frac{y^2 + 1}{x^2 + 1} \)