Math 128a, problem set 06
Outline due: Wed Oct 13
Due: Mon Oct 18
Last revision due: Mon Nov 15

Problems to be done, but not turned in: (Ch. 6) 7, 15, 17, 23, 27, 33, 35; (Ch. 7) 1, 3, 5, 9, 15.
Fun: (Ch. 6) 44.

Problems to be turned in:

1. Does there exist an automorphism \( \varphi : \mathbb{Z}_{71} \to \mathbb{Z}_{71} \) such that \( \varphi(13) = 25 \)? If so, describe all such \( \varphi \) as precisely as possible, with proof; if not, prove that no such \( \varphi \) exists.

2. Consider the groups \( U(16) \), \( U(20) \), and \( U(24) \). For any two of them that you think are not isomorphic, prove that they are not isomorphic.

3. Find three groups \( G, H, K \) of order 120 such that \( G \not\approx H \), \( H \not\approx K \), and \( G \not\approx K \). Prove your result.

4. Consider the group \( D_6 \), using our standard notation.
   (a) Let \( K = \{e, F_{12}\} = \langle F_{12} \rangle \). List all of the left cosets of \( K \) and all of the right cosets of \( K \).
   (b) Let \( H = \{e, R_{120}, R_{240}\} = \langle R_{120} \rangle \). List all of the left cosets of \( H \) and all of the right cosets of \( H \). Do you see any significant qualitative differences between this example and the previous one? Explain.

5. (Ch. 7) 6.

6. Let \( G \) be a group, and let \( H \) and \( K \) be subgroups of \( G \) such that \( |H| = 60 \) and \( |K| = 76 \). What are the possibilities for the order of \( H \cap K \)? Generalize.

7. (a) Let \( G \) be a group such that every nontrivial element of \( G \) has order 2. Prove that \( G \) is abelian.
   (b) Now let \( G \) be a group of order 8. Prove that if \( G \) is not abelian, then \( G \) must have an element of order 4.