Sample exam 3, Spring 2002

1. (10 points) Let $A$ be an $n \times n$ matrix. Define what it means for $v$ to be an eigenvector of $A$.

2. (12 points) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 2 & 1 \\ 3 & 6 & c & 3 \\ 4 & 8 & 12 & 10 \end{bmatrix}$. Find the value of $\det A$, in terms of $c$, and determine all values of $c$ for which $A$ is invertible. Show all your work.

3. (12 points) Choosing $A$, and performing a calculation, we see that

$$A = \begin{bmatrix} 2 & -1 & -5 & 2 & 10 \\ 1 & -1 & -4 & 3 & 4 \\ 1 & -2 & -7 & 2 & 7 \\ 1 & 0 & -1 & 1 & 4 \end{bmatrix}, \quad \text{ref}(A) = \begin{bmatrix} 1 & 0 & -1 & 0 & 5 \\ 0 & 1 & 3 & 0 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$ 

(a) Find a basis for Col $A$. No explanation necessary, but show all your work.

(b) Find a basis for Null $A$. No explanation necessary, but show all your work.

4. (14 points) Let $A = \begin{bmatrix} 3 & 8 & 8 & 0 \\ -3 & -4 & 0 & 3 \\ 3 & 3 & -1 & -3 \\ -2 & 2 & 8 & 5 \end{bmatrix}$. You may take it as given that the characteristic polynomial of $A$ is $(t + 1)^2(t - 2)(t - 3)$. (In other words, do not spend time checking this fact.)

Exactly one of the eigenspaces of $A$ has dimension 2. Find the eigenvalue $\lambda$ of this eigenspace, and find a basis for this eigenspace. Show all your work.

5. (8 points) (T/F) Let $V$ be the set of all $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in $\mathbb{R}^4$ such that $x_1 - x_2 + x_3 + 2x_4 = 0$ and $5x_2 - 3x_3 + x_4 = 0$. Then $V$ is a subspace of $\mathbb{R}^4$.

6. (8 points) (T/F) Let $V$ be the set of all $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in $\mathbb{R}^2$ such that $|x_1| = |x_2|$. Then $V$ is a subspace of $\mathbb{R}^2$.

7. (8 points) (T/F) It is possible that there exists a subspace $V$ of $\mathbb{R}^5$, and vectors $u_1, u_2, u_3, u_4, u_5$ in $V$, such that $\dim V = 4$ and $\{u_1, u_2, u_3, u_4, u_5\}$ is a basis for $V$.

8. (8 points) (T/F) Let $u, v, w$ be nonzero vectors in $\mathbb{R}^4$, and let $V = \text{Span}\{u, v, w\}$. Then $\dim V$ must be 3.

9. Let $\{v_1, v_2, v_3\}$ be a basis for a subspace $V$ of $\mathbb{R}^5$.

(a) (10 points) Is $\{3v_1, 2v_2, v_1 - v_2\}$ a basis for $V$? Explain why or why not.

(b) (10 points) Is $\{v_1 - v_2 + v_3, v_2 - v_3, -v_3\}$ a basis for $V$? Explain why or why not.