Problems to be done, but not turned in: (3.1) 7, 11; (3.2) 6, 9, 13; (3.3) 5, 7, 9, 10, 13, 17.

Problems to be turned in:

1. Let $V$ be a vector space, let $W_1$ and $W_2$ be subspaces of $V$, and let

$$U = \{ v \in V \mid v = w_1 + w_2 \text{ for some } w_1 \in W_1, w_2 \in W_2 \}.$$ 

Prove that $U$ is a subspace of $V$.

2. (3.2) 11(b).

3. (3.2) 14.

4. Let $V$ be a vector space, and let $v, w, x$ be vectors in $V$ such that $v + w + x = 0$. Let $W_1 = \text{span}\{v, w\}$, and let $W_2 = \text{span}\{w, x\}$. Must it be true that $W_1 = W_2$? Prove or give a counterexample.

5. (3.3) 15.

6. Let $V$ be a vector space, and let $v_1, v_2, v_3$ be nonzero vectors in $V$.

   (a) Give an example of a vector space $V$ and $v_1, v_2, v_3 \in V$ such that $v_3 \in \text{span}\{v_1 - v_2, v_2 - v_3\}$.

   (b) Now suppose that $\{v_1, v_2, v_3\}$ is linearly independent. Is it possible that $v_3 \in \text{span}\{v_1 - v_2, v_2 - v_3\}$? Give an example or prove that it is not possible.