Math-112 Vector Calculus  Test-1 (100 Points)  Last Name: _______________
(Saleem)   Fall 2012    First Name: _____________

(You are allowed to use a 3”x 5” card of notes and a calculator. Show all work.)

##1. (5 Points) Using the “addition of vectors” write expressions for the missing “?” vectors:

\[
\begin{align*}
? &= \text{___________________} \\
? &= \text{___________________}
\end{align*}
\]

##2. (15 Points) An equilateral triangle is attached to a square of side 1, as shown. O is the origin.

(a) Compute the following vectors in the form \( ai + bj + ck \). Simplify completely:

Vector OB = \[
\begin{bmatrix}
3 \\
2 \\
1
\end{bmatrix}
\]

Vector OD = \[
\begin{bmatrix}
1 \\
-2 \\
-3
\end{bmatrix}
\]

Vector OC = \[
\begin{bmatrix}
1 \\
-2 \\
-3
\end{bmatrix}
\]

Vector OE = \[
\begin{bmatrix}
1 \\
-2 \\
-3
\end{bmatrix}
\]

(a) Compute the dot product of OD and OE

(b) Using dot products and part (a) prove that angle DOE is 15 degrees. Use a calculator.

##3. (10 Points) The area of a parallelogram with sides \( \mathbf{u} \) and \( \mathbf{v} \) is given by \( |\mathbf{u} \times \mathbf{v}| \). Find the area if 

\[
\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}
\]

Leave square roots in the answer.
(a) **Draw** the vector field \( \mathbf{F} = y \mathbf{i} - x \mathbf{j} \) by first completing the following table:

<table>
<thead>
<tr>
<th>(x,y)</th>
<th>(1,0)</th>
<th>(0,1)</th>
<th>(-1,0)</th>
<th>(0,-1)</th>
<th>(1,2)</th>
<th>(-1,2)</th>
<th>(2,-1)</th>
<th>(-2,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{F} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Compute the line integral of vector field \( \mathbf{F} \) over \( C \), where \( C \) is the segment of the curve \( y = \sqrt{x} \) from the point \((0,0)\) to \((9,3)\).

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**5. (15 Points)** Consider a vector field \( \mathbf{F} = (1 - 7y^2) \mathbf{i} + (4y - 14xy) \mathbf{j} \), which is known to be conservative such that, \( \mathbf{F} = \nabla \phi \) where \( \phi = x + 2y^2 - 7xy^2 \). Use well-known theorems to compute the following line integrals: (only half credit for direct integration)

(a) \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is circle, \( x^2 + y^2 = 16 \), clockwise.

(b) \( \int_Q \mathbf{F} \cdot d\mathbf{r} \) where \( Q \) is upper half of the circle, \( x^2 + y^2 = 16 \), clockwise.
6. (5 Points) Simplify the vector product completely by showing all work: \((\mathbf{a} \times \mathbf{b}) \times (\mathbf{b} \times \mathbf{c})\)

7. (10 Points) (a) With \(\mathbf{r} = x \hat{i} + y \hat{j}\), compute the cross product, \(\mathbf{r} \times d\mathbf{r}\)

(b) For a simple closed curve \(C\) in the \(XY\) plane, apply Green’s Theorem to simplify \(\oint_C \mathbf{r} \times d\mathbf{r}\)

8. (20 Points) Find the flux of \(\mathbf{F} = \frac{2}{z} \hat{k}\) over one-eighth of the surface of a sphere (lying in the first octant). The equation for the sphere is \(x^2 + y^2 + z^2 = 25\). For reference, spherical coordinates are given here: \(x = \rho \sin \phi \cos \theta; \quad y = \rho \sin \phi \sin \theta; \quad z = \rho \cos \phi\). Show all work and a picture.