A non-homogeneous equation and a particular solution are given. Find a general solution for the equation.

\[ y'' = 2y' - y + 2e^x, \quad y_p(x) = x^2 e^x. \]

**SOLUTION:** It suffices to find the general solution \( y_h \) to the corresponding homogeneous equation \( y'' - 2y' + y = 0 \). Then \( y_{\text{general}} = y_h + y_p \) will be the general solution to the non-homogeneous equation.

The associated auxiliary equation \( r^2 - 2r + 1 = 0 \) has a double root \( r = 1 \), so

\[ y_h = C_1 e^x + C_2 xe^x \]

(not \( C_1 e^x + C_2 e^x \)). Therefore,

\[ y_{\text{general}} = C_1 e^x + C_2 xe^x + x^2 e^x. \]