4.1, ex. 4 The characteristic equation is \( s^2 + 4s + 13 = 0 \), which yields \( s_{1,2} = -2 \pm 3i \). Thus
\[
y_h = c_1 e^{-2t} \cos t 3t + c_2 e^{-2t} \sin 3t.
\]
We seek \( y_p \) in the form \( ce^{-t} \). Substituting into the ODE we get \( ce^{-t} - 4ce^{-t} + 13ce^{-t} = e^{-t} \), so \( c = 1/10 \). Therefore,
\[
y_{\text{general}} = c_1 e^{-2t} \cos t 3t + c_2 e^{-2t} \sin 3t + \frac{1}{10} e^{-t}. \quad \square
\]

4.1, ex. 20 We seek for \( y_p \) in the form \( y_p = K \), where \( K \) is a constant. Substituting into the ODE, we obtain \( qK = c \). Thus \( y_p = c/q \). \quad \square

4.2, ex. 6 The characteristic equation is \( s^2 + 6s + 8 = 0 \), which yields \( s_{1,2} = -3, 2 \). Thus
\[
y_h = c_1 e^{-3t} + c_2 e^{-2t}.
\]
Complexifying we obtain the ODE
\[
y'' + 6y' + 8y = -4e^{3it}.
\]
We seek for its particular solution in the form \( y_c = ae^{3it} \), for some complex constant \( a \). Substituting into the complexified ODE we obtain
\[
a = \frac{4}{-1 + 18i} = \frac{4 + 72i}{325}.
\]
Thus
\[
y_c = \frac{4 + 72i}{325} (\cos 3t + i \sin 3t) = \frac{1}{325} (4 \cos 3t - 72 \sin 3t).
\]
Taking the real part, we obtain
\[
y_p = \text{Re}(y_c) = \frac{4}{325} \cos 3t. \quad \square