2.1, ex. 20 (a) Suppose \( y(t) = \cos \beta t \) is a solution of
\[
y'' + \frac{k}{m} y = 0.
\]
Then
\[
0 = (\cos \beta t)'' + \frac{k}{m} \cos \beta t = -\beta^2 \cos \beta t + \frac{k}{m} \cos \beta t,
\]
so we must have \(-\beta^2 + k/m = 0\), i.e.,
\[
\beta = \pm \sqrt{\frac{k}{m}}.
\]
(b) The initial condition satisfied by this solution is
\[
(y(0), v(0)) = (1, 0),
\]
where \( v = dy/dt = -\beta \sin \beta t \).

(c) Since \( \cos \) has minimal period \( 2\pi \), it follows that the minimal period of \( \cos \beta t \) is \( 2\pi/\beta = 2\pi/\sqrt{k/m} \).

(d) Since
\[
y^2 + \left(\frac{v}{\beta}\right)^2 = \cos^2 \beta t + \sin^2 \beta t = 1,
\]
for all \( t \), this solution curve is the ellipse centered at the origin with semiaxes 1 and \( \beta \). □

2.2, ex. 6 (a) The vector field is
\[
\mathbf{F}(x, y) = \begin{bmatrix} x \\ 2y \end{bmatrix}.
\]

(c)

(d)
(e) There is a unique equilibrium at the origin. All other solutions diverge from the origin as $t \to \infty$. □

2.2, ex. 20 Since

$$\mathbf{F}(y, v) \cdot \begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} v \\ -y \end{bmatrix} \cdot \begin{bmatrix} y \\ v \end{bmatrix} = vy - yv = 0,$$

the vector field $\mathbf{F}$ at any point $(y, v)$ is orthogonal to the radius (or position) vector $(y, v)$. Therefore, $\mathbf{F}$ is tangent to circles centered at the origin. □