2.4, ex. 14 (a) Since $Y_1$ is a solution, we have

$$\frac{dY_1}{dt}(T) = AY_1(T),$$

for every $T$, in particular, for $T = t + t_0$. Thus:

$$\frac{dY_2}{dt} = \frac{d}{dt} Y_1(t + t_0)$$

$$= \frac{dY_1}{dt}(t + t_0)$$

$$= AY_1(t + t_0)$$

$$= AY_2(t),$$

so $Y_2$ is also a solution.

(b) The curves $Y_1$ and $Y_2$ are the same as sets. The only difference between them is that $Y_2(t)$ is $t_0$-seconds “ahead” of $Y_1(t)$.

3.1, ex. 18 The given ODE is equivalent to the system

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = 0.$$  

(a) The general solution of the second equation is $v(t) = C$, where is a constant, or $v(t) = v_0$, where $v_0$ is the initial value for $v$.

(b) Substituting $v(t) = v_0$ into the first equation and integrating, we obtain $y(t) = v_0t + K$, where $K$ is a constant. Setting $t = 0$, it is easy to see that $K = y_0$, the initial value for $y$.

(c) Since $v(t)$ is constant, the solution starting at $(y_0, v_0)$ lies on the horizontal straight line $v = v_0$. If $v_0 > 0$, the solution moves to the right, if $v_0 < 0$, it moves to the left. If $v_0 = 0$, then $y(t)$ is also constant. This means that the $y$-axis consists of equilibria. See Fig.1.
Figure 1. Sec. 3.1, ex. 18: the phase portrait.

3.1, ex. 24 (a) Since

$$\frac{dY_1}{dt} = \frac{d}{dt} \begin{bmatrix} 0 \\ e^t \end{bmatrix} = \begin{bmatrix} 0 \\ e^t \end{bmatrix} = A \begin{bmatrix} 0 \\ e^t \end{bmatrix} = AY_1(t),$$

where

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix},$$

$Y_1(t)$ is a solution. Similarly,

$$\frac{dY_2}{dt} = \frac{d}{dt} \begin{bmatrix} e^{2t} \\ e^{2t} \end{bmatrix} = \begin{bmatrix} 2e^{2t} \\ 2e^{2t} \end{bmatrix} = A \begin{bmatrix} e^{2t} \\ 2e^{2t} \end{bmatrix} = AY_2(t),$$

so $Y_2(t)$ is also a solution.

(b) We seek the solution to the given initial-value problem in the form

$$Y(t) = c_1 Y_2(t) + c_2 Y_2(t),$$

for some constants $c_1, c_2$. Setting $t = 0$, we obtain

$$\begin{bmatrix} -2 \\ -1 \end{bmatrix} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_2 \\ c_1 + c_2 \end{bmatrix}.$$
Thus $c_2 = -2$ and $c_1 + c_2 = -1$. Solving by substitution, we obtain $c_1 = -1 - c_2 = 1$. Therefore, the unique solution to the given initial-value problem is

\[ Y(t) = Y_1(t) - 2Y_2(t) = \begin{bmatrix} 0 \\ e^t \end{bmatrix} - 2 \begin{bmatrix} e^{2t} \\ e^{2t} \end{bmatrix} = \begin{bmatrix} -2e^{2t} \\ e^t - 2e^{2t} \end{bmatrix}. \]