3.4, ex. 2 If $B$ has $\lambda = -2 + 5i$ as an eigenvalue and

$$Y_0 = \begin{bmatrix} 1 \\ 4 - 3i \end{bmatrix}$$

as an eigenvector, then the general solution of $dY/dt = BY$ is a linear combination of the real part $Y_{re}(t)$ and the imaginary part $Y_{im}(t)$ of $e^{\lambda t}Y_0$. We have

$$e^{\lambda t}Y_0 = e^{(-2+5i)t} \begin{bmatrix} 1 \\ 4 - 3i \end{bmatrix} = e^{-2t}(\cos 5t + i \sin 5t) \begin{bmatrix} 1 \\ 4 - 3i \end{bmatrix} = e^{-2t} \begin{bmatrix} \cos 5t + i \sin 5t \\ (4 \cos 5t + 3 \sin 5t) + i(4 \sin 5t - 3 \cos 5t) \end{bmatrix}$$

Therefore,

$$Y_{re}(t) = \begin{bmatrix} e^{-2t} \cos 5t \\ e^{-2t}(4 \cos 5t + 3 \sin 5t) \end{bmatrix}, \quad Y_{im}(t) = \begin{bmatrix} e^{-2t} \sin 5t \\ e^{-2t}(4 \sin 5t - 3 \cos 5t) \end{bmatrix},$$

and the general solution is

$$Y(t) = c_1 Y_{re}(t) + c_2 Y_{im}(t).$$

3.4, ex. 16 The trace of

$$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

is $T = 2a$ and its determinant is $D = a^2 + b^2$, so the characteristic equation is

$$\lambda^2 - 2a\lambda + a^2 + b^2.$$

Therefore, the eigenvalues of $A$ are

$$\lambda_{1,2} = \frac{2a \pm \sqrt{4a^2 - 4(a^2 + b^2)}}{2} = a \pm ib,$$

hence complex.

3.5, ex. 22 (a) The first system can be written as

$$\frac{dx}{dt} = 2y, \quad \frac{dy}{dt} = 0.$$

Hence $y(t) = y_0$. Substituting into the first equation, we obtain

$$\frac{dx}{dt} = 2y_0,$$
so \[ x(t) = 2y_0 t + x_0. \]

Thus the general solution is
\[ Y(t) = \begin{bmatrix} 2y_0 t + x_0 \\ y_0 \end{bmatrix}. \]

(b) The second system can be written as
\[
\frac{dx}{dt} = -2y, \\
\frac{dy}{dt} = 0.
\]

Hence \( y(t) = y_0. \) Substituting into the first equation, we obtain
\[
\frac{dx}{dt} = -2y_0,
\]

so \[ x(t) = -2y_0 t + x_0. \]

Thus the general solution is
\[ Y(t) = \begin{bmatrix} -2y_0 t + x_0 \\ y_0 \end{bmatrix}. \]