1. (25 points) Consider the following system of ODEs:
\[
\frac{dx}{dt} = 3x + 4y \\
\frac{dy}{dt} = x.
\]
(a) Compute the eigenvalues.
(b) For each eigenvalue, compute the associated eigenvectors.
(c) Compute the general solution.
(d) Sketch the phase portrait and classify it.

Solution:
2. (25 points) Consider the following system of ODEs:

\[
\frac{dx}{dt} = 2y \\
\frac{dy}{dt} = -2x.
\]

(a) Compute the eigenvalues.

(b) Find the solution starting at the point \((1, 0)\) and sketch it.

(d) Sketch the phase portrait and classify it.

Solution:
3. **(25 points)** Consider the following system of ODEs:

\[
\frac{dY}{dt} = AY, \quad A = \begin{bmatrix} -5 & -2 \\ -1 & -4 \end{bmatrix}.
\]

(a) Compute the eigenvalues.
(b) For each eigenvalue, compute the associated eigenvectors.
(c) Compute the general solution.
(d) Sketch the phase portrait and classify it.

**Solution:**
4. (25 points) Find the solution to the following initial-value problem:

\[
\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 13y = 0, \quad y(0) = 1, \quad y'(0) = -4.
\]

Solution: