Find the general solution of
\[ \frac{d^2y}{dt^2} + 7 \frac{dy}{dt} + 10y = e^{-2t}. \]

**Solution:** The characteristic equation of the associated homogeneous equation is
\[ s^2 + 7s + 10 = 0. \]
Its solutions are \( s_1, s_2 = -5, -2 \). Thus the general solution to the homogeneous equation is
\[ y_h(t) = c_1 e^{-5t} + c_2 e^{-2t}. \]

Since \( e^{-2t} \) (the right-hand side of the non-homogeneous equation) is part of \( y_h(t) \), we seek a particular solution in the form
\[ y_p(t) = Ate^{-2t}. \]

Substituting, we obtain
\[(4t - 4)Ae^{-2t} + 7A(1 - 2t)e^{-2t} + 10Ate^{-2t} = e^{-2t}.\]

Dividing by \( e^{-2t} \) and simplifying, we obtain \( 3A = 1 \), so \( A = 1/3 \). Therefore, \( y_p(t) = \frac{t}{3} e^{-2t} \) and the general solution is
\[ y(t) = y_h(t) + y_p(t) = c_1 e^{-5t} + c_2 e^{-2t} + \frac{t}{3} e^{-2t}. \]